

The Age of Information: Real-Time Status Updating by Multiple Sources

Roy D. Yates and Sanjit K. Kaul

Abstract

We examine multiple independent sources providing status updates to a monitor through simple queues. We formulate a status-age timeliness metric and derive a general result for the average status age that is applicable to a wide variety of multiuser service systems. An alternate technique for evaluating status age based on stochastic hybrid systems is also introduced. For first-come first-served and two types of last-come first-served systems with Poisson arrivals and exponential service times, we find the region of feasible average status ages for multiple updating sources. We then use these results to characterize how a service facility can be shared among multiple updating sources.

Index Terms

Age of information, queueing systems, random processes, communication networks

I. INTRODUCTION

Increasingly ubiquitous connectivity to communication networks and availability of portable devices have engendered a host of applications in which sources – people and environmental sensors – send updates of their status to interested recipients. These include news and weather reports and updates by individuals on Twitter about what is keeping them busy, updates by environmental sensors [1], and vehicular status (position, velocity, acceleration) updates that can assist drivers of nearby vehicles in an intelligent transportation system [2]. These applications need status updates at one or more monitors to be *as timely as possible*; however, this is typically constrained by limited network resources.

Roy Yates is with WINLAB and the ECE Department, Rutgers University, NJ, USA, e-mail: ryates@winlab.rutgers.edu.

Sanjit Kaul is with Wireless Systems Lab, IIIT-Delhi, India, e-mail: skkaul@iiitd.ac.in.

This work was presented in part at the 2012 Conference on Information Sciences and Systems and the 2012 IEEE International Symposium on Information Theory. This work was supported by NSF award CCF-1422988.

For example, consider various sensors (location, acceleration, tire pressure, etc.) in a vehicle generating status updates which the in-vehicle radio delivers to other vehicles in vicinity or other networked monitoring systems. The update packets are queued while they wait to be serviced by the car radio. The packet currently being serviced by the radio waits for medium access and transmission before it is received by other cars. Note that each sensor in the car may be a source or the car may aggregate a collection of sensor measurements into a status update message that is transmitted as a single packet. The packet service time will depend on the wireless channel and may or may not incorporate retransmissions due to channel errors and backoff due to the activity of other wireless transmitters. While system models that incorporate these effects can be arbitrarily complex, we observe that optimal updating policies are not well understood even in the simple setting of M/M/1 queues.

Maintaining the timeliness of data and state information in a network is a problem that has appeared in many forms, including, for example, data freshness in warehouses [3] and web caches [4], periodic updating of real time databases [5], and route caches in ad hoc networks [6]. However, no consistent analytic methodology has emerged. This paper focuses on an *age of information (AoI)* timeliness metric as a basis for the evaluation and design of status update systems.

When a monitor's most recently received update at time t is time-stamped $u(t)$, the *status update age* or simply the *age*, is the random process $\Delta(t) = t - u(t)$ and the AoI is the average $\Delta(t)$. The monitor's requirement of timely updating corresponds to small AoI. While AoI is an application-independent metric that permits evaluation of the network performance, separate from application-specific metrics that may be too complex to employ in the design of the network. However, AoI can also be useful in specific applications by designing the communication network to meet statistical requirements, such as expected value and variance, of the age process. For example, if a status updating system is reporting sample values of a Wiener process $X(t)$ with variance αt [7], then the monitor's MMSE estimate of $X(t)$ given the status age $\Delta(t)$ is $\hat{X}(t) = X(t - \Delta(t))$. The variance of this estimate is $\alpha\Delta(t)$.

Traditionally, network performance has been characterized by tradeoffs in rate, delay, throughput and loss. The data rate can be increased, but this induces additional delay in lossless systems or increased packet dropping in lossy systems. Furthermore, comparisons between lossless and lossy networks are generally problematic. By contrast, we will see that AoI is fundamentally

different; the age metric enables direct comparison of lossless and lossy systems. Moreover, the goal of timely updating is neither the same as maximizing the throughput or utilization of the communication system, nor of ensuring that generated status updates are received with minimum delay. Utilization is maximized when sensors send updates as fast as possible. However, this can lead to the monitor receiving delayed updates because the status messages become backlogged in the communication system. Instead, we will see that sources can minimize their AoI by optimizing their updating rates in response to the available system resources.

We further observe that it may also be desirable to redesign systems to facilitate timely updating. A basic property of the first-come first-served (FCFS) queue model is that new update messages can be queued behind outdated messages that were generated earlier. This can be viewed as an undesirable consequence of protocol layering or of the hardware design. However, among all status update packets in the wireless interface, the transmission of the youngest packet will minimize the status age at the monitor. Moreover, under the assumption that a status update carries the Markov state of the source, the transmission of the youngest status update obviates the need for transmission of the older outdated packets in the queue. Thus it is desirable to implement a lossy last-come first-served (LCFS) queueing discipline in which a new status update packet will preempt any previously queued update packets and this preempted packet will be discarded. This example motivates the LCFS analysis in Section IV.

A. Prior Work and Related Applications

This paper expands on our analyses of status age in single-source single-server queues [8], the M/M/1 LCFS queue with preemption in service [9], and the M/M/1 FCFS system with multiple sources [10]. Other contributions to AoI analysis have also appeared recently. To evaluate AoI for a single source sending updates through a network cloud [11] or through an M/M/2 server [12], [13], out-of-order packet delivery was the key analytical challenge. A related (and generally more tractable) metric, peak age of information (PAoI), was introduced in [14]. Properties of PAoI have also been studied in [15] for an FCFS M/G/1 multiclass queue. In [14], [16], the authors analyzed AoI and PAoI for M/M/1/1 and M/M/1/2 queues that discard arriving updates if the system is full and also for a third queue, dubbed M/M/1/2*, in which an arriving update would preempt a waiting update. In this work, the M/M/1/2* queue is called the M/M/1 LCFS-W (Last Come First Served with preemption only in Waiting) queue and here we extend AoI

results to a LCFS-W system with multiple sources.

Most recently, optimality properties of a Last Generated First Served (LGFS) service discipline when updates arrive out of order are identified in [17], packet deadlines are found to improve AoI in [18], AoI in the presence of errors is evaluated in [19], and LCFS with non-memoryless gamma-distributed service times is considered in [20]. There have also been recent studies of energy-constrained updating [21]–[23] in which updates are submitted to the server with knowledge of the server state.

In addition to these queue-theoretic AoI analyses, the theme of ensuring “freshness” has also appeared in various application areas, including that of networks, real time databases and warehousing.

In [24], we look at minimizing the age of status updates sent by vehicles over a carrier-sense multiple access (CSMA) network. A local minimum for the age can be approached using gradient descent; however, it is not known if this is a global minimum and is only seen to exist in simulations. In [25], we show that allowing nodes to piggyback other nodes’ status updates can lead to a smaller age.

For safety-related intelligent transport system applications, an *Awareness Quality* metric [26] captures how fine and up-to-date the application information is. The authors observe that default metrics like throughput and delay are unable to capture awareness and propose *Update Delay*, which is the elapsed time between application updates. In [27], the authors propose to use an oldest packet drop mechanism instead of a tail drop policy to reduce the delay of the received information, via beacons, in vehicular networks.

In [3], the authors want to maximize the freshness of data in warehouses to meet user demands. They estimate the queue length and delay at the warehouse staging area where updates wait before they are committed to the warehouse database. Experiments lead them to conclude that small queues are desirable.

Web caching reduces the latency in returning a web page to a client. However, unless refreshed often enough, a cache will return stale web pages. The refresh rate is limited by the finite time it takes for a cache to be updated after the page has been updated at the server. In [4] the authors propose an architecture that limits the “*degree of staleness*” of a cache. Our work, for fairly simple descriptions of the time it takes to update a cache, answers how often the cache must be refreshed such that its age is minimized.

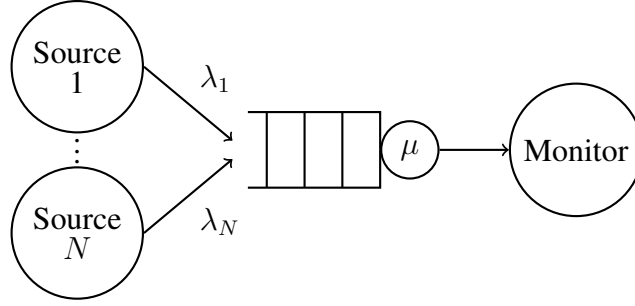


Fig. 1. Independent sources send through a queue to a monitor.

In [5] the authors look at periodic transactions updating real time databases. Each transaction updates the database with data that is associated with a deadline relative to when it is generated. In their work, there is no assumed limit on available processing power (service rate). The objective is to find the combination of update period and deadline such that all transactions complete before their deadlines, thus ensuring the freshness of data while minimizing the CPU utilization.

Ad hoc networking protocols typically use cached routes to forward packets to their destinations. In [6] the authors propose a mechanism that avoids propagation of stale route information. To avoid the overhead associated with periodic broadcasts of new route information, their method uses an epoch numbering system that helps network nodes to reject older information. In [28] the authors consider the issue of frequency of hello messages in ad-hoc networks. The frequency must not be so large as to congest the network but also not too small that the nodes have stale information.

Finally, data dissemination in sensor networks has been looked at under varied constraints. For example, in works like [29] and [30] the authors consider energy efficient dissemination of state in sensor networks. More frequent updates lead to greater energy consumption and smaller sensor lifetime. Our work suggests strategies that a sensor, when awake, can use to minimize the average age of its status updates.

B. Summary of Results

This work is based on the system depicted in Figure 1 in which a server delivers the updates of N sources to a monitor. In Section II, we derive Theorem 1, a general result that describes the AoI Δ_i for each source i in terms of the stationary properties of the interarrival times and system times of delivered source i updates. We then apply Theorem 1 to M/M/1 systems in which

updates packets arrive as Poisson processes and have memoryless service times. The service rate is μ for updates from any source. This is sufficient to model systems in which the updating sources have identical fixed length update packets but heterogeneous timeliness requirements.

For M/M/1 systems, Section III uses Theorem 1 to derive the AoI in an N -source FCFS system. Section IV continues the analysis of Poisson updaters with AoI derivations for a pair of lossy N -source M/M/1 LCFS systems. The first is LCFS with preemption of the packet in service (LCFS-S.) The second LCFS system (called LCFS-W) permits preemption only in waiting. For each of these systems, we derive simple closed form expressions for the AoI of each source in terms of the offered loads of the N updating sources.

We first analyze AoI in the LCFS-S queue using Theorem 1. This analysis bears similarity to the FCFS analysis in Section III. We next take advantage of the small state space of the LCFS-S system to introduce an analysis technique, namely stochastic hybrid systems (SHS) [31], that has not been previously applied to status updating systems. Although the SHS approach may not appear to be simple, it provides a systematic (and largely mechanical) procedure for the calculation of AoI in queues with memoryless service. In particular, for the LCFS-W system, AoI analysis based on Theorem 1 requires careful partitioning of cases and becomes quite complex. Thus, we instead employ the SHS method to derive the AoI of the LCFS-W system with multiple sources in Section IV-D. In Section V, we compare the lossless FCFS and lossy LCFS systems. A short conclusion follows in Section VI.

Although the M/M/1 queue models that we examine are often too simple to describe practical networks, status age is a new metric that is not well understood. We start here with these simple models to develop an understanding of the age of information in shared queues, in order to go on to characterize status age in more complex practical systems.

II. STATUS UPDATE AGE: PRELIMINARY ANALYSIS

Figure 2 shows a sample variation of age $\Delta_i(t)$, for source i as a function of time t , at the monitor. Without loss of generality, assume that we begin observing at $t = 0$ when the queue is empty and the age is $\Delta_i(0)$. The first status update of source i is timestamped t_1 and is followed by updates timestamped t_2, t_3, \dots, t_n . The status age of source i at the monitor increases linearly in time in the absence of any updates and is reset to a smaller value when an update is received. Update j of source i , generated at time t_j , finishes service and is received by the monitor at

time t'_j . At t'_j , the age $\Delta_i(t'_j)$ at the monitor is reset to the age $T_j = t'_j - t_j$ of the received status update. The age T_j is also the system time of update packet j . Thus the age function $\Delta_i(t)$ exhibits the sawtooth pattern shown in Figure 2. The time average age of the status updates is the area under the age graph in Figure 2 normalized by the time interval of observation.

Over an interval $(0, \mathcal{T})$, the average age is

$$\langle \Delta_i \rangle_{\mathcal{T}} = \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} \Delta_i(t) dt. \quad (1)$$

For simplicity of exposition, the length of the observation interval is chosen to be $\mathcal{T} = t'_n$, as depicted in Figure 2. We decompose the area defined by the integral (1) into the sum of the polygon area \tilde{Q}_1 , the trapezoidal areas Q_j for $j \geq 2$ (Q_2 and Q_n are highlighted in the figure), and the triangular area of width T_n over the time interval (t_n, t'_n) . From Figure 2, we see that Q_j can be calculated as the difference between the area of the isosceles triangle whose base connects the points t_{j-1} and t'_j and the area of the isosceles triangle with base connecting the points t_j and t'_j . Defining

$$Y_j = t_j - t_{j-1} \quad (2)$$

to be the interarrival time of update j , it follows that

$$Q_j = \frac{1}{2}(T_j + Y_j)^2 - \frac{1}{2}T_j^2 = Y_j T_j + Y_j^2/2. \quad (3)$$

With $N_i(\mathcal{T}) = \max\{n | t_n \leq \mathcal{T}\}$ denoting the number of source i updates by time \mathcal{T} , this decomposition, along with some rearrangement, yields the time-average age

$$\langle \Delta_i \rangle_{\mathcal{T}} = \frac{\tilde{Q}}{\mathcal{T}} + \frac{(N_i(\mathcal{T}) - 1) \sum_{j=2}^{N_i(\mathcal{T})} Q_j}{N_i(\mathcal{T}) - 1} \quad (4)$$

where $\tilde{Q} = \tilde{Q}_1 + T_n^2/2$. We observe that the age contribution \tilde{Q} represents a boundary effect that is finite with probability 1, so the first term in (4) will vanish as \mathcal{T} grows.

We will say that a status updating system is stationary and ergodic if Y_j and T_j are stationary sequences with marginal distributions identical to Y and T respectively, and as $\mathcal{T} \rightarrow \infty$,

$$\frac{N_i(\mathcal{T})}{\mathcal{T}} \rightarrow \frac{1}{\mathbb{E}[Y]}, \quad \text{and} \quad \frac{\sum_{j=2}^{N_i(\mathcal{T})} Q_j}{N_i(\mathcal{T}) - 1} \rightarrow \mathbb{E}[Q] \quad (5)$$

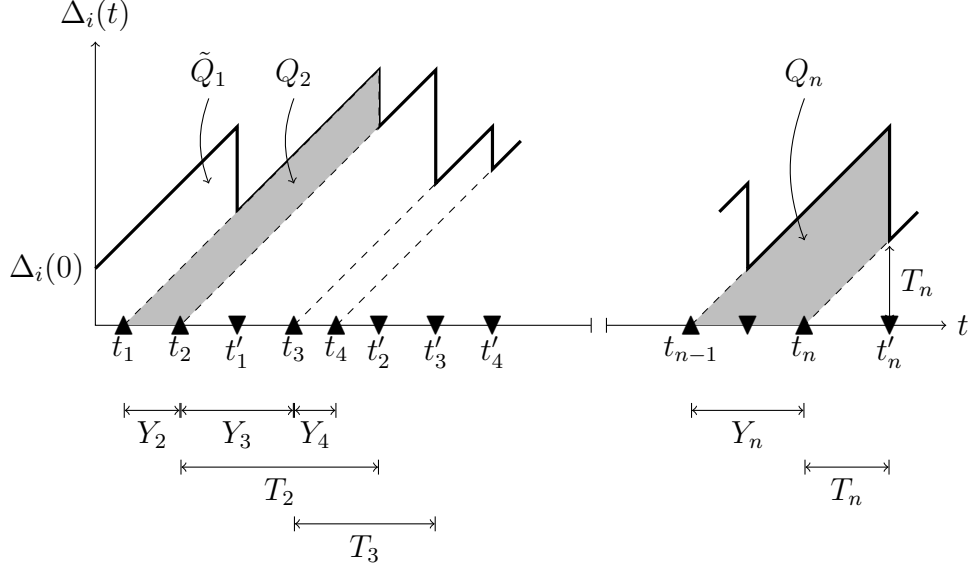


Fig. 2. Example change in status update age at a monitor for a system with a FCFS queue. Updates from source 1 arrive at times marked \blacktriangle and are received at the monitor at times marked \blacktriangledown .

with probability 1. For such systems, the AoI of source i is

$$\Delta_i = \lim_{\mathcal{T} \rightarrow \infty} \langle \Delta_i \rangle_{\mathcal{T}} \quad (6)$$

and (4) implies the next claim.

Theorem 1: For a stationary ergodic status updating system in which Y is the interarrival time between delivered source i updates and T is the system time of such a delivered packet, the AoI for source i is

$$\Delta_i = \frac{E[Q]}{E[Y]} = \frac{E[YT] + E[Y^2]/2}{E[Y]}.$$

We note that Theorem 1 is more akin to a bookkeeping identity such as Little's Law in that sufficient conditions for the ergodicity of the age process are not explicitly provided but can be verified for most reasonably-designed service systems. As a consequence, Theorem 1 can be applied to a broad class of service systems, including both lossless FCFS systems as well as lossy LCFS systems in which updates are preempted and discarded. Furthermore, it makes no specific assumptions regarding other traffic that might share the queue with the update packets of source i .

We emphasize that Y is the interarrival time between *delivered* updates of source i , and T is the

system time of a *delivered* update. These somewhat cumbersome definitions are a consequence of the generality of the approach. For example, Section III employs Theorem 1 to evaluate a work-conserving M/M/1 FCFS system in which the Y_j are independent identically distributed (iid) exponential interarrival times and the T_j are the packet system times. On the other hand, Section IV uses Theorem 1 to analyze the LCFS-S that supports preemption of the packet in service. In this system, packet j refers to the j th packet that completes service and is delivered to the monitor. There may be an arbitrarily large number of update packets that arrive between packets $j - 1$ and j that are preempted and discarded before completing service.

III. M/M/1 FIRST-COME FIRST-SERVED

In prior work [8], we analyzed M/M/1 FCFS queues serving the status updates of a single source. In that work, it was shown that the average status age for an M/M/1 queue with arrival rate λ , service rate μ and offered load $\rho = \lambda/\mu$ is given by

$$\Delta = \frac{1}{\mu} \left[\frac{\rho^2}{1 - \rho} + 1 + \frac{1}{\rho} \right]. \quad (7)$$

The average age Δ in (7) is minimized at $\rho^* \approx 0.53$. In this section, we generalize this result to an N source system.

Following Theorem 1, the system time of a source i update packet is $T = W + S$, where W and S are the respective waiting and service times. Since S is independent of Y , it follows that $E[YT] = E[YW] + E[Y]E[S]$. We note that $E[S] = 1/\mu$ and that the rate λ_i Poisson arrival process implies $E[Y] = 1/\lambda_i$, and $E[Y^2] = 2/\lambda_i^2$. It follows from Theorem 1 that

$$\Delta_i = \lambda_i E[YW] + \frac{1}{\mu} + \frac{1}{\lambda_i}. \quad (8)$$

The expectation $E[YW]$ is nontrivial because Y and W are negatively correlated; a large interarrival time Y can allow the queue to empty, yielding a small waiting time W . Furthermore, the expected value $E[YW]$ also depends on the aggregate other-source updating load

$$\rho_{-i} = \rho - \rho_i = \sum_{j \neq i} \rho_j. \quad (9)$$

Evaluation of $E[YW]$ is provided in Appendix A in the proof of the following lemma.

Lemma 1:

$$E[YW] = \frac{1}{\mu^2} \left[\frac{\rho_i(1 - \rho\rho_{-i})}{(1 - \rho)(1 - \rho_{-i})^3} + \frac{\rho_{-i}}{\rho_i(1 - \rho_{-i})} \right].$$

Applying Lemma 1 to (8) yields the following result:

Theorem 2: N sources with offered loads ρ_1, \dots, ρ_N and total load $\rho = \sum_j \rho_j$ at a rate μ M/M/1 FCFS queue have average ages $\Delta_1, \dots, \Delta_N$ such that

$$\Delta_i = \frac{1}{\mu} \left[\frac{\rho_i^2(1 - \rho\rho_{-i})}{(1 - \rho)(1 - \rho_{-i})^3} + \frac{1}{1 - \rho_{-i}} + \frac{1}{\rho_i} \right].$$

We note that Theorem 2 reduces to the single source result (7) when $\rho_i = \rho$ and $\rho_{-i} = 0$.

IV. M/M/1 LAST COME FIRST SERVED

We will explore two possibilities under LCFS. First, under LCFS *with* preemption-in-service (LCFS-S), we allow the new packet to preempt the packet currently in service. Second, under LCFS with *preemption only in waiting* (LCFS-W) queue discipline, the new status packet replaces any older status packet *waiting* in the queue; however, it has to *wait* for any packet currently in service to finish. In this work, preemption is assumed to be source agnostic. We will allow a source's packet to be preempted by that of another source. For the case of M/M/1 systems, the main result is summarized here:

Theorem 3: N sources with offered loads ρ_1, \dots, ρ_N and at a rate μ M/M/1 LCFS queue with total load $\rho = \sum_{i=1}^N \rho_i$ have average ages $\Delta_1, \dots, \Delta_N$ such that

(a) with preemption allowed in service (LCFS-S),

$$\Delta_i = \frac{1}{\mu} (1 + \rho) \frac{1}{\rho_i},$$

(b) and with preemption allowed only in waiting (LCFS-W),

$$\Delta_i = \frac{1}{\mu} \left[\alpha_W(\rho) + \left(1 + \frac{\rho^2}{1 + \rho} \right) \frac{1}{\rho_i} \right]$$

where

$$\alpha_W(\rho) = \frac{(1 + \rho + \rho^2)^2 + 2\rho^3}{(1 + \rho + \rho^2)(1 + \rho)^2}.$$

We note that while $\alpha_W(\rho)$ is a ratio of fourth order polynomials, direct calculation will verify that

$$0.837 < \alpha_W(\rho) < 1.09, \quad \rho \geq 0. \quad (10)$$

We defer the proof of Theorem 3 to Sections IV-B, IV-C and IV-D.

A. Discussion

We note that for a single source with $\rho_1 = \rho$, Theorem 3(b) can be shown to reduce to the AoI of the M/M/2/2* queue, as given in [16, Equation (65)]. We also note Theorem 3(b) corrects an error in [9, Equation (23)]. In the context of a single-source system, this error was identified and explained in [16, Appendix]. That explanation serves to highlight how easily mistakes can be made in AoI analysis, even in simple memoryless-service systems. At the conclusion of [16, Appendix], the authors argue “In the LCFS system with preemption we expect that, for very large arrival rates, the age would increase without bound, as no packet finishes service.” We note that this speculation is contrary to the result of Theorem 3(a), which in the special case of a single-source with $\rho_1 = \rho = \lambda/\mu$, shows that the average age approaches $1/\mu$ as $\lambda \rightarrow \infty$.¹

In this work, we provide in Section IV-B a derivation of Theorem 3(a) using the method in [9], but with some algebraic simplifications that went previously unrecognized. As it will be based on Theorem 1, this proof will be conceptually similar to that for FCFS systems. However, as this approach has been viewed with skepticism in [16], we provide in Section IV-C a wholly different derivation of Theorem 3(a). In this alternate proof, we model the LCFS-S system as a stochastic hybrid system (SHS) [31] in which the queue state (i.e., the type of update packet in service) evolves as a discrete-state Markov chain while state variables that track the age process vary continuously. While this second type of proof is not essential, it serves to introduce the SHS method in a system with a (relatively small) state space.

In Section IV-D, we go on to employ a SHS to derive the AoI of the LCFS-W queue as given in Theorem 3(b). With preemption only in waiting, the Markov state of the queue must track

¹For the single-source LCFS-S system, this asymptotic result is a consequence of memoryless service. With fixed service rate μ and arrival rate $\lambda \rightarrow \infty$, the server is always occupied and (because the service is memoryless) the queue departure rate approaches μ . That is, the queue inter-departures approach a Poisson process of rate μ . While the fraction $\mu/(\lambda + \mu)$ of those updates that complete service goes to zero, those that do complete service have system time T that approaches zero. In this limiting case, the interarrival time Y of a delivered update becomes an exponential (μ) random variable. In the context of Theorem 1 and the sawtooth age process in Figure 2, $E[TY] \rightarrow 0$, $E[Y^2] \rightarrow 2/\mu$ and $E[Y] \rightarrow 1/\mu$.

the source of both the update in service and the update in waiting. With this increase in the size of the queue state space, analysis methods based on Theorem 1 become overly complex.

B. LCFS With Preemption In Service: Analysis

In this system, a packet arrival preempts the packet currently in service, if any. The number of packets in such a system is at most 1. To analyze this system, we start with Theorem 1. As shown in Figure 2, update packets generated by source i at time instants t_j are those updates that *complete* service and $Y_j = t_j - t_{j-1}$ is the time between such arrivals. The service time (and also system time) of this j th packet is T_j .

In order to calculate Δ_i , let D_j (see Figure 3) be the time interval between the departures $j - 1$ and j . This interval starts with an idle period and may see zero or more arrivals of other sources, some of which may complete service, while others are preempted. Any arrivals of the given source during D_j , other than arrival j , are preempted. Thus the interval D_j consists of one or more blocks of the server being idle followed by it being busy. Note that if the system consists of just one source, then D_j consists of just one block, which starts with the idle period that follows the departure of $j - 1$. This idle period is followed by the server busy period that ends in departure j . Figure 3 shows D_j , which contains a random L number of blocks. The figure shows blocks 1 and L . A block k , say of length B_k , consists of an idle period of length X'_k followed by a busy period of length S_k . We have

$$D_j = \sum_{k=1}^L B_k = \sum_{k=1}^L (X'_k + S_k). \quad (11)$$

Note that packet j arrives during S_L and then spends time T_j in service.

We will now calculate the terms $E[Y]$, $E[Y^2]$ and $E[YT]$ in Theorem 1 in terms of D_j and T_j . Consider the interval Y_j , for any j . We observe from Figure 3 that

$$Y_j + T_j = T_{j-1} + D_j. \quad (12)$$

Because $T =^{st} T_{j-1} =^{st} T_j$, $Y =^{st} Y_j$, and $D =^{st} D_j$,

$$E[Y] = E[Y_j] = E[D_j] = E[D]. \quad (13)$$

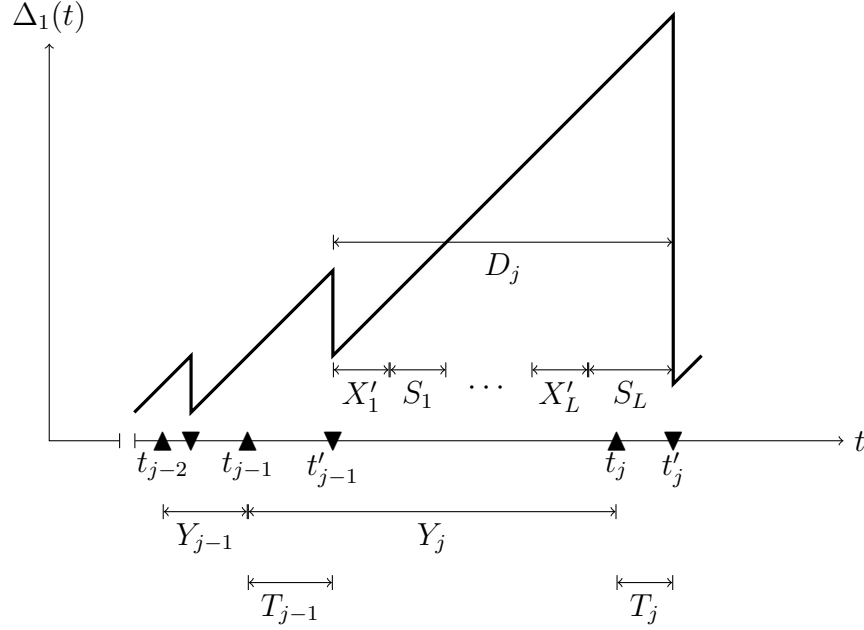


Fig. 3. Example change in update age of source i under LCFS *with* preemption in service. On the time axis, updates from source i arrive at times marked \blacktriangle and are received at the monitor at times marked \blacktriangledown .

Note that Y_j and T_j are independent. Thus (13) implies

$$\mathbb{E}[Y_j T_j] = \mathbb{E}[Y_j] \mathbb{E}[T_j] = \mathbb{E}[Y] \mathbb{E}[T] = \mathbb{E}[D] \mathbb{E}[T]. \quad (14)$$

Furthermore, since D_j and T_{j-1} are also independent, it also follows from (12) that

$$\text{Var}[Y_j] + \text{Var}[T_j] = \text{Var}[T_{j-1}] + \text{Var}[D_j]. \quad (15)$$

It then follows from (13) that $\mathbb{E}[Y^2] = \mathbb{E}[D^2]$. This fact, combined with (13) and (14), simplify Theorem 1 to

$$\Delta_i = \mathbb{E}[T] + \frac{\mathbb{E}[D^2]}{2\mathbb{E}[D]}. \quad (16)$$

The remainder of the proof of Theorem 3(a), specifically the calculation of the moments in (16), appears in Appendix B.

C. LCFS With Preemption In Service: SHS Analysis

We now provide an alternate derivation of Theorem 3(a) for the age of the LCFS-S system. While this alternate proof is not necessary, it provides a relatively simple setting in which to introduce a method of calculating average using a stochastic hybrid systems (SHS) approach [31]. In Section IV-D, we go on to use the SHS method for the LCFS-W (with preemption only in waiting) queue, which we will see has a substantially larger state space.

In the SHS method [31], there is a discrete state $q(t) \in \mathcal{Q}$ describing the occupancy of the server and a continuous state $\mathbf{x}(t)$ that describes the continuous-time evolution of the system age. In discrete transitions from state q to q' , the continuous state can have discontinuous jumps from \mathbf{x} to \mathbf{x}' . We refer the reader to [31] for additional background on SHS.

Without loss of generality, we assume a two-source system and we solve for the average age Δ_1 of source 1. In terms of the N source system, source 2 represents the composition of all other sources. This LCFS-S system has three discrete states $q(t) = q \in \mathcal{Q} = \{0, 1, 2\}$ such that $q = 0$ indicates that the server is idle and $q \in \{1, 2\}$ denotes the type of update packet in service.

The continuous state is $\mathbf{x}(t) = [x_0(t), x_1(t)]$ where $x_0(t)$ is the current age $\Delta_1(t)$ of the source 1 process, and $x_1(t)$ is the reduction in $\Delta_1(t)$ that will occur if the packet-in-service is delivered. We note that $x_1(t)$ does not evolve continuously as it is given by the current age at the time an update packet from source 1 is admitted to the queue. Instead, in a discrete state transition from q to q' at time t' , $x_1(t')$ can make discontinuous jumps as well as inducing downward jumps in $x_0(t')$. However, while the system remains in a discrete state q , the evolution of $\mathbf{x}(t)$ is given by

$$\frac{d\mathbf{x}(t)}{dt} = [1, 0]. \quad (17)$$

The interpretation of (44) is that the age $\Delta_1(t) = x_0(t)$ increases at unit rate with time t while $x_1(t)$ is unchanged in the absence of a state transition.

A Markov chain for the discrete state $q(t)$ is shown in Figure 4. We use \mathcal{L} to denote the set of links l that correspond to transitions of the discrete state Markov chain. Link l from q to q' indicates that transitions in state q to state q' occur at exponential rate $\lambda^{(l)}$ as given in Table I. Table I also specifies for each link l the transition/reset maps $\phi_l(q, \mathbf{x}) = (q', \mathbf{x}')$, where (q, \mathbf{x}) and (q', \mathbf{x}') are the system states the instant before and after the transition. For example, link $l = 1$ marks the arrival of a source 1 update at an empty queue. With this arrival at time t ,

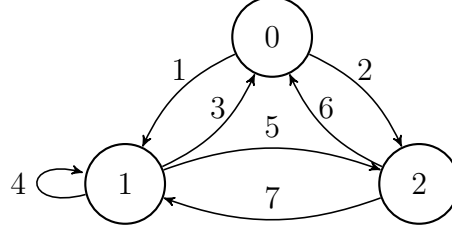


Fig. 4. The SHS Markov chain for updates of source 1 in the LCFS-S system. The transition rates and transition/reset maps for links $l = 1, \dots, 7$ are shown in Table I.

$x'_0(t) = x_0(t)$ is unchanged by the transition because the arrival does not yield an age reduction until it departs. However, this packet immediately goes into service and if it departs at a time $\hat{t} > t$,

$$x'_0(\hat{t}) = x_0(\hat{t}) - x_1(\hat{t}) = x_0(\hat{t}) - x'_1(t) = x_0(\hat{t}) - x_0(t). \quad (18)$$

This corresponds to the age $\Delta_1(\hat{t})$ being reduced at the time of the update delivery by $\Delta_1(t)$. The first two equalities in (18) follow from the definition of $\mathbf{x}(t)$ and its evolution. The last equality follows from the fact that the reduction in age is equal to the age at the time t of the arrival. If this is unclear, we refer to Figure 2 where we observe that the size of the downward jump of the sawtooth age function at the time of a type 1 departure is precisely equal to the status age at the time of the arrival of the corresponding update.

We note that unlike an ordinary continuous-time Markov chain, the SHS includes self-transitions in which the discrete state is unchanged because a reset occurs in the continuous state. Specifically, in state 1, the self-transition of link 4 marks the arrival of a source 1 update packet that preempts the source 1 packet in service. This leaves the discrete state $q(t)$ unchanged, but the more recent timestamp of the new source 1 update resets $x_1(t)$.

The SHS method uses test functions whose expected values converge to steady-state quantities of interest, such as the average age. For $\bar{q} \in \mathcal{Q}$ and binary vectors $m = (m_0, m_1)$, we will employ test functions

$$\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = \mathbf{x}^m \delta_{\bar{q}, q}, \quad (19)$$

where $\delta_{\bar{q}, q}$ denotes the Kronecker delta function and \mathbf{x}^m is shorthand for the monomial

$$\mathbf{x}^m = [x_0, x_1]^m = x_0^{m_0} x_1^{m_1}. \quad (20)$$

link l	$q \rightarrow q'$	$\lambda^{(l)}(q)$	$\phi_l(q, \mathbf{x}) = (q', \mathbf{x}')$
1	$0 \rightarrow 1$	$\lambda_1 \delta_{0,q}$	$(1, [x_0, x_0])$
2	$0 \rightarrow 2$	$\lambda_2 \delta_{0,q}$	$(2, [x_0, 0])$
3	$1 \rightarrow 0$	$\mu \delta_{1,q}$	$(0, [x_0 - x_1, 0])$
4	$1 \rightarrow 1$	$\lambda_1 \delta_{1,q}$	$(1, [x_0, x_0])$
5	$1 \rightarrow 2$	$\lambda_2 \delta_{1,q}$	$(2, [x_0, 0])$
6	$2 \rightarrow 0$	$\mu \delta_{2,q}$	$(0, [x_0, 0])$
7	$2 \rightarrow 1$	$\lambda_1 \delta_{2,q}$	$(1, [x_0, x_0])$

TABLE I
TABLE OF TRANSITIONS FOR THE MARKOV CHAIN IN FIGURE 4.

For $m = (0, 0)$, the expected value of the test function $\psi_{\bar{q}}^{(0,0)}(q, \mathbf{x})$ yields the Markov state probabilities

$$\pi_{\bar{q}}(t) = \mathbb{P}[q(t) = \bar{q}] = \mathbb{E}[\delta_{\bar{q}, q(t)}] = \mathbb{E}[\psi_{\bar{q}}^{(0,0)}(q(t), \mathbf{x}(t))]. \quad (21)$$

Associated with a SHS is a mapping $\Psi \rightarrow L\Psi$ known as the extended generator. Because of the simple form of (17) and the time-invariant transition/reset maps in Table I, the extended generator of the status age SHS is given by

$$(L\psi)(q, \mathbf{x}) = \frac{\partial \psi(q, \mathbf{x})}{\partial x_0} + \sum_{l \in \mathcal{L}} (\psi(\phi_l(q, x)) - \psi(q, x)) \lambda^{(l)}(q). \quad (22)$$

Each test function $\psi(q(t), \mathbf{x}(t))$ must satisfy

$$\frac{d \mathbb{E}[\psi(q(t), \mathbf{x}(t))]}{dt} = \mathbb{E}[(L\psi)(q(t), \mathbf{x}(t))]. \quad (23)$$

We also observe from (19) that

$$\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = \begin{cases} x_0 \delta_{\bar{q}, q} & m = (1, 0), \\ x_1 \delta_{\bar{q}, q} & m = (0, 1). \end{cases} \quad (24)$$

Hence we define the corresponding variables

$$v_{0\bar{q}}(t) = \mathbb{E}[\psi_{\bar{q}}^{(1,0)}(q(t), \mathbf{x}(t))] = \mathbb{E}[x_0(t) \delta_{\bar{q}, q(t)}], \quad (25a)$$

$$v_{1\bar{q}}(t) = \mathbb{E}[\psi_{\bar{q}}^{(0,1)}(q(t), \mathbf{x}(t))] = \mathbb{E}[x_1(t) \delta_{\bar{q}, q(t)}]. \quad (25b)$$

Based on the Markov chain in Figure 4 and the transition map in Table I, the SHS method applies (22) and (23) to each function $\psi(q, \mathbf{x}) = \psi_{\bar{q}}^{(m)}(q, \mathbf{x})$. This yields a set of first order linear differential equations for the variables $\pi_{\bar{q}}(t)$ and $v_{i\bar{q}}(t)$ for $\bar{q} \in \mathcal{Q}$ and $i = 0, 1$. In principle, we will obtain 9 equations, but we will see that the Markov structure will simplify the calculations. From these differential equations, we will see that as $t \rightarrow \infty$, $\pi_{\bar{q}}(t)$ will converge to the stationary probability $\pi_{\bar{q}}^*$ and that each $v_{i\bar{q}}(t) = \mathbb{E}[x_i(t)\delta_{\bar{q},q(t)}]$ will converge to a steady state value $v_{i\bar{q}}^*$. Since

$$\Delta_1(t) = \sum_{\bar{q} \in \mathcal{Q}} x_0(t) \delta_{\bar{q},q(t)}, \quad (26)$$

these steady state values yield the average age

$$\Delta_1 = \lim_{t \rightarrow \infty} \mathbb{E}[\Delta_1(t)] = \lim_{t \rightarrow \infty} \sum_{\bar{q} \in \mathcal{Q}} \mathbb{E}[x_0(t) \delta_{\bar{q},q(t)}] = \sum_{\bar{q} \in \mathcal{Q}} v_{0\bar{q}}^*. \quad (27)$$

From (22), we calculate $L\psi_{\bar{q}}^{(m)}$ for each m and $\bar{q} \in \mathcal{Q}$:

$$L\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = m_0 x_0^{m_0-1} x_1^{m_1} \delta_{\bar{q},q} + \mu \Lambda_{\bar{q}}^{(m)}(q, \mathbf{x}) \quad (28)$$

where

$$\Lambda_{\bar{q}}^{(m)}(q, \mathbf{x}) = \frac{1}{\mu} \sum_{l \in \mathcal{L}} \left(\psi_{\bar{q}}^{(m)}(\phi_l(q, \mathbf{x})) - \psi_{\bar{q}}^{(m)}(q, \mathbf{x}) \right) \lambda^{(l)}(q). \quad (29)$$

We recall that $\rho_i = \lambda_i/\mu$ and $\rho = \rho_1 + \rho_2$ and we define

$$\beta = 1 + \rho, \quad \beta_i = 1 + \rho_i. \quad (30)$$

Using $\Lambda_{\bar{q}}^{(m)}$ as shorthand for $\Lambda_{\bar{q}}^{(m)}(q, \mathbf{x})$ (which is shorthand for $\Lambda_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))$), it follows from Table I that evaluation of (29) for $\bar{q} = 0, 1, 2$ yields

$$\Lambda_0^{(m)} = [x_0 - x_1, 0]^m \delta_{1,q} + [x_0, 0]^m \delta_{2,q} - \rho \mathbf{x}^m \delta_{0,q}, \quad (31a)$$

$$\Lambda_1^{(m)} = \rho_1 [x_0, x_0]^m (\delta_{0,q} + \delta_{1,q} + \delta_{2,q}) - \beta \mathbf{x}^m \delta_{1,q}, \quad (31b)$$

$$\Lambda_2^{(m)} = \rho_2 [x_0, 0]^m (\delta_{0,q} + \delta_{1,q}) - \beta_1 \mathbf{x}^m \delta_{2,q}. \quad (31c)$$

Applying (23) to $\psi(q(t), \mathbf{x}(t)) = \psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))$, it follows from (28) that

$$\frac{d}{dt} \mathbb{E}[\psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))] = m_0 \mathbb{E}[\mathbf{x}^{(m_0-1, m_1)}(t) \delta_{\bar{q}, q(t)}] + \mu \mathbb{E}[\Lambda_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))]. \quad (32)$$

From (21) and (32) with $m = (0, 0)$, the state probability vector $\pi(t) = [\pi_0(t) \ \pi_1(t) \ \pi_2(t)]'$ has component derivatives $\dot{\pi}_{\bar{q}}(t) = d\pi_{\bar{q}}(t)/dt$ that satisfy

$$\dot{\pi}_{\bar{q}}(t) = \mu \mathbb{E}[\Lambda_{\bar{q}}^{(0,0)}(q(t), \mathbf{x}(t))]. \quad (33)$$

Furthermore, for $m = (0, 0)$, (31) reduces to

$$\Lambda_0^{(0,0)} = -\rho \delta_{0,q(t)} + \delta_{1,q(t)} + \delta_{2,q(t)}, \quad (34a)$$

$$\Lambda_1^{(0,0)} = \rho_1 (\delta_{0,q(t)} + \delta_{1,q(t)} + \delta_{2,q(t)}) - \beta \delta_{1,q(t)}, \quad (34b)$$

$$\Lambda_2^{(0,0)} = \rho_2 (\delta_{0,q(t)} + \delta_{1,q(t)}) - \beta_1 \delta_{2,q(t)}. \quad (34c)$$

Thus (33) and (34) imply $\pi(t)$ satisfies $\dot{\pi}(t) = \mu \mathbf{R} \pi(t)$ with

$$\mathbf{R} = \begin{bmatrix} -\rho & 1 & 1 \\ \rho_1 & -1 - \rho_2 & \rho_1 \\ \rho_2 & \rho_2 & -1 - \rho_1 \end{bmatrix}. \quad (35)$$

Applying $\dot{\pi}(t) = 0$, the $\pi_i(t)$ converge to the stationary probabilities π_i^* satisfying $\mathbf{R} \pi^* = 0$ and $\sum_{i=0}^2 \pi_i^* = 1$. These stationary probabilities are

$$\begin{bmatrix} \pi_0^* & \pi_1^* & \pi_2^* \end{bmatrix}' = (1 + \rho)^{-1} \begin{bmatrix} 1 & \rho_1 & \rho_2 \end{bmatrix}'. \quad (36)$$

For $m = (1, 0)$ and $m = (0, 1)$, (25) and (32) imply $v_{0\bar{q}}(t)$ and $v_{1\bar{q}}(t)$ have derivatives satisfying

$$\dot{v}_{0\bar{q}}(t) = \mathbb{E}[\delta_{\bar{q}, q(t)}] + \mu \mathbb{E}[\Lambda_{\bar{q}}^{(1,0)}(q(t), \mathbf{x}(t))], \quad (37a)$$

$$\dot{v}_{1\bar{q}}(t) = \mu \mathbb{E}[\Lambda_{\bar{q}}^{(0,1)}(q(t), \mathbf{x}(t))]. \quad (37b)$$

With $m = (1, 0)$, taking the expectation of (31) yields

$$\mathbb{E}[\Lambda_0^{(1,0)}] = -\rho v_{00} + v_{01} + v_{02} - v_{11}, \quad (38a)$$

$$\mathbb{E}[\Lambda_1^{(1,0)}] = \rho_1 v_{00} - \beta_2 v_{01} + \rho_1 v_{02}, \quad (38b)$$

$$\mathbb{E}[\Lambda_2^{(1,0)}] = \rho_2 v_{00} + \rho_2 v_{01} - \beta_1 v_{02}. \quad (38c)$$

With $m = (0, 1)$, taking the expectation of (31) yields

$$\mathbb{E}[\Lambda_0^{(0,1)}] = -\rho v_{10}, \quad (39a)$$

$$\mathbb{E}[\Lambda_1^{(0,1)}] = \rho_1(v_{00} + v_{01} + v_{02}) - \beta v_{11}, \quad (39b)$$

$$\mathbb{E}[\Lambda_2^{(0,1)}] = -\beta v_{12}. \quad (39c)$$

Combining (37a) for $\bar{q} = 0, 1, 2$ with (38) yields

$$\dot{v}_{00} = \mu(-\rho v_{00} + v_{01} + v_{02} - v_{11}) + \pi_0(t), \quad (40a)$$

$$\dot{v}_{01} = \mu(\rho_1 v_{00} - \beta_2 v_{01} + \rho_1 v_{02}) + \pi_1(t), \quad (40b)$$

$$\dot{v}_{02} = \mu(\rho_2 v_{00} + \rho_2 v_{01} - \beta_1 v_{02}) + \pi_2(t). \quad (40c)$$

Similarly, for $\bar{q} = 0, 1, 2$, combining (37b) with (39) yields

$$\dot{v}_{10} = -\mu \rho v_{10}, \quad (41a)$$

$$\dot{v}_{11} = \mu[\rho_1(v_{00} + v_{01} + v_{02}) - \beta v_{11}], \quad (41b)$$

$$\dot{v}_{12} = -\mu \beta v_{12}. \quad (41c)$$

We see that (41a) implies $\lim_{t \rightarrow \infty} v_{10}(t) = 0$. In fact, $v_{10}(t) = \mathbb{E}[x_1(t)\delta_{0,q(t)}] = 0$ because $x_1(t)\delta_{0,q(t)} = 0$ since $x_1(t)$ is set to 0 when the system enters state $q = 0$ but $\delta_{0,q(t)} = 0$ when $q(t) \neq 0$. In the same way, (41c) reflects that $v_{12}(t) = 0$. We refer to such variables that are always zero as *trivial*.

In general, as $t \rightarrow \infty$, $\pi(t) \rightarrow \pi^*$, $\dot{v}_{ij}(t) \rightarrow 0$ and $v_{ij}(t) \rightarrow v_{ij}^*$. Gathering the nontrivial v_{ij} and setting $\dot{v}_{ij} = 0$, (40) and (41) imply

$$\begin{bmatrix} \pi_0^* \\ \pi_1^* \\ \pi_2^* \\ 0 \end{bmatrix} = \mu \begin{bmatrix} \rho & -1 & -1 & 1 \\ -\rho_1 & \beta_2 & -\rho_1 & 0 \\ -\rho_2 & -\rho_2 & \beta_1 & 0 \\ \rho_1 & \rho_1 & \rho_1 & -\beta \end{bmatrix} \begin{bmatrix} v_{00}^* \\ v_{01}^* \\ v_{02}^* \\ v_{11}^* \end{bmatrix}. \quad (42)$$

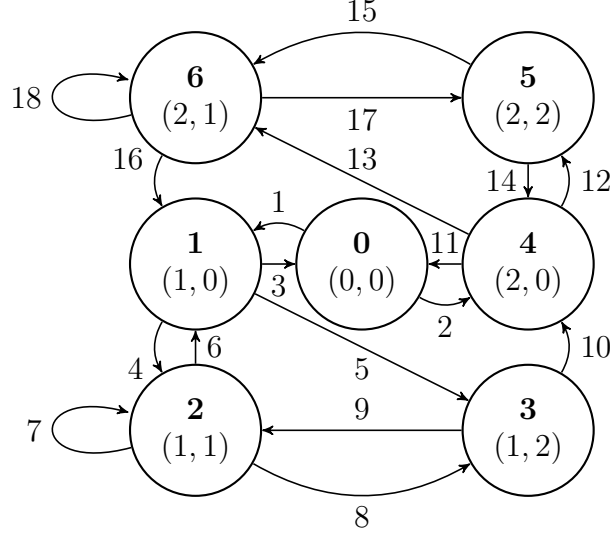


Fig. 5. The Markov chain for the discrete state $q(t)$ for the LCFS system.

It follows from (36) and (42) that

$$v_{00}^* = \frac{1}{\mu(1+\rho)} \left[\frac{1+\rho_2}{\rho_1} + \frac{1}{1+\rho} \right], \quad (43a)$$

$$v_{01}^* = \frac{1}{\mu(1+\rho)} \left[1 + \rho + \frac{\rho_1}{1+\rho} \right], \quad (43b)$$

$$v_{02}^* = \frac{1}{\mu(1+\rho)} \left[\frac{\rho_2(1+\rho)}{\rho_1} + \frac{\rho_2}{1+\rho} \right]. \quad (43c)$$

Theorem 3(a) then follows from (27) and (43).

D. LCFS With Preemption Only In Waiting: SHS Analysis

Following the SHS approach introduced in Section IV-C, we now model the LCFS-W system as a stochastic hybrid system. As we did in Section IV-C, we assume a two-source system and we solve for the average age Δ_1 of source 1. In terms of the N source system, source 2 represents the composition of all other sources.

The LCFS-W system with discrete states $q \in \mathcal{Q} = \{0, 1, \dots, 6\}$ is shown in Figure 5. In the figure, we also mark each state by the pair (i, j) such that $i \in \{0, 1, 2\}$ denotes the type of update packet that is occupying the server and $j \in \{0, 1, 2\}$ denotes the type of update packet waiting for service. In this convention, $i = 0$ indicates that no update packet is in service. Similarly, $j = 0$ indicates that no update packet is waiting.

The continuous state is $\mathbf{x}(t) = [x_0(t), x_1(t), x_2(t)]$ where $x_0(t)$ is the current age $\Delta_1(t)$ of the source 1 process, $x_1(t)$ is the reduction in $\Delta_1(t)$ that will occur when the packet-in-service is delivered and $x_2(t)$ is the future reduction in age (for source 1) that would result from the currently waiting update packet being delivered. As in the LCFS-S system, only $x_0(t)$ evolves continuously; $x_1(t)$ and $x_2(t)$ make jumps only in a discrete state transition. In each discrete state q , the evolution of $\mathbf{x}(t)$ is given by

$$\frac{d\mathbf{x}(t)}{dt} = [1, 0, 0]. \quad (44)$$

Just as in the LCFS-S system, the interpretation of (44) is that the age $\Delta_1(t) = x_0(t)$ increases at unit rate with time t while $x_1(t)$ and $x_2(t)$ are unchanged in the absence of a state transition.

In Figure 5, a link l from node q to q' indicates that transitions in state q to state q' occur at exponential rate $\lambda^{(l)}$ as given in Table II. Table II also specifies the transition/reset maps $\phi_l(q, \mathbf{x}) = (q', \mathbf{x}')$. For $x_0(t)$ and $x_1(t)$, the maps are similar to those for the LCFS-S system. The additional complications involve how $x_2(t)$ modifies $x_1(t)$ when an update completes service (and the waiting update goes into service) and how $x_2(t)$ is modified when the waiting update is preempted.

We note that this SHS Markov chain also includes self transitions. Specifically, in states 2 and 6, the self-transitions of links 7 and 18 mark the arrival of a source 1 update packet that supplants the waiting source 1 packet. This leaves the discrete state $q(t)$ unchanged, but the more recent timestamp of the new source 1 waiting packet resets $x_2(t)$. In state 6, this source 1 arrival resets $x_2(t)$ to $x_0(t)$ since if this new arrival eventually completes service at some future time t' , then at time t' , $\Delta_1(t')$ would be reduced by $x_0(t) = \Delta_1(t)$. Somewhat similarly, in state 2 at time t , the source 1 packet in service will (upon delivery at a future time t') reduce $\Delta_1(t')$ by $x_1(t)$. The new source 1 update packet corresponding to link 7 transition will, if it eventually completes service at time t' reduce $\Delta_1(t')$ by $x_0(t) - x_1(t)$.

For $\bar{q} \in \mathcal{Q}$ and binary vectors $m = (m_0, m_1, m_2)$, the LCFS-W system will employ the test functions

$$\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = \mathbf{x}^m \delta_{\bar{q}, q}, \quad (45)$$

link l	$q \rightarrow q'$	$\lambda^{(l)}(q)$	$\phi_l(q, \mathbf{x}) = (q', \mathbf{x}')$
1	$0 \rightarrow 1$	$\lambda_1 \delta_{0,q}$	$(1, [x_0, x_0, 0])$
2	$0 \rightarrow 4$	$\lambda_2 \delta_{0,q}$	$(4, [x_0, 0, 0])$
3	$1 \rightarrow 0$	$\mu \delta_{1,q}$	$(0, [x_0 - x_1, 0, 0])$
4	$1 \rightarrow 2$	$\lambda_1 \delta_{1,q}$	$(2, [x_0, x_1, x_0 - x_1])$
5	$1 \rightarrow 3$	$\lambda_2 \delta_{1,q}$	$(3, [x_0, x_1, 0])$
6	$2 \rightarrow 1$	$\mu \delta_{2,q}$	$(1, [x_0 - x_1, x_2, 0])$
7	$2 \rightarrow 2$	$\lambda_1 \delta_{2,q}$	$(2, [x_0, x_1, x_0 - x_1])$
8	$2 \rightarrow 3$	$\lambda_2 \delta_{2,q}$	$(3, [x_0, x_1, 0])$
9	$3 \rightarrow 2$	$\lambda_1 \delta_{3,q}$	$(2, [x_0, x_1, x_0 - x_1])$
10	$3 \rightarrow 4$	$\mu \delta_{3,q}$	$(4, [x_0 - x_1, 0, 0])$
11	$4 \rightarrow 0$	$\mu \delta_{4,q}$	$(0, [x_0, 0, 0])$
12	$4 \rightarrow 5$	$\lambda_2 \delta_{4,q}$	$(5, [x_0, 0, 0])$
13	$4 \rightarrow 6$	$\lambda_1 \delta_{4,q}$	$(6, [x_0, 0, x_0])$
14	$5 \rightarrow 4$	$\mu \delta_{5,q}$	$(4, [x_0, 0, 0])$
15	$5 \rightarrow 6$	$\lambda_1 \delta_{5,q}$	$(6, [x_0, 0, x_0])$
16	$6 \rightarrow 1$	$\mu \delta_{6,q}$	$(1, [x_0, x_2, 0])$
17	$6 \rightarrow 5$	$\lambda_2 \delta_{6,q}$	$(5, [x_0, 0, 0])$
18	$6 \rightarrow 6$	$\lambda_1 \delta_{6,q}$	$(6, [x_0, 0, x_0])$

TABLE II
TABLE OF TRANSITIONS FOR THE MARKOV CHAIN IN FIGURE 5.

where \mathbf{x}^m denotes the monomial

$$\mathbf{x}^m = [x_0, x_1, x_2]^m = x_0^{m_0} x_1^{m_1} x_2^{m_2}. \quad (46)$$

For $m = (0, 0, 0)$, the test function $\psi_{\bar{q}}^{(0,0,0)}(q, \mathbf{x})$ yields the Markov state probabilities

$$\pi_{\bar{q}}(t) = \mathbb{P}[q(t) = \bar{q}] = \mathbb{E}[\psi_{\bar{q}}^{(0,0,0)}(q(t), \mathbf{x}(t))]. \quad (47)$$

The extended generator $\Psi \rightarrow L\Psi$ is still given by (22) and each test function must still satisfy (23).

Since (45) implies

$$\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = \begin{cases} x_0 \delta_{\bar{q},q} & m = (1, 0, 0), \\ x_1 \delta_{\bar{q},q} & m = (0, 1, 0), \\ x_2 \delta_{\bar{q},q} & m = (0, 0, 1), \end{cases} \quad (48)$$

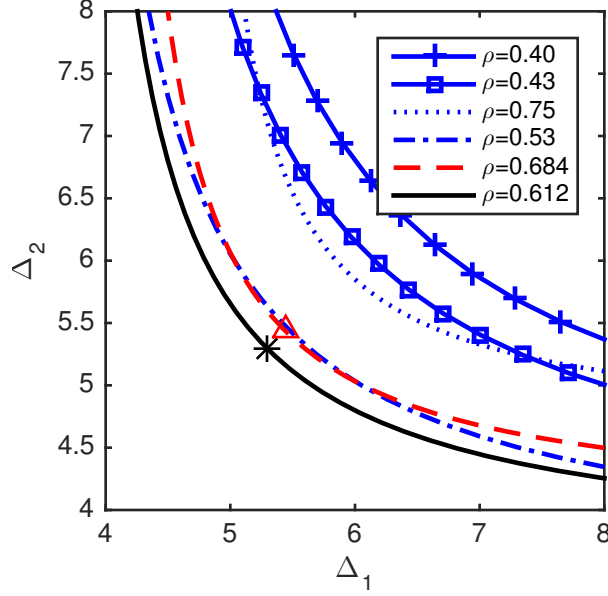


Fig. 6. Age contours (Δ_1, Δ_2) for fixed total load $\rho_1 + \rho_2 = \rho$ for two sources sharing a rate $\mu = 1$ FCFS M/M/1 queue. The minimum sum age point marked * in the lower left is achieved by $\rho_1 = \rho_2 = 0.306$. The point Δ marks the Nash equilibrium age pair achieved by unilateral optimization.

we define the corresponding variables

$$v_{0\bar{q}}(t) = E[\psi_{\bar{q}}^{(1,0,0)}(q(t), \mathbf{x}(t))] = E[x_0(t)\delta_{\bar{q},q(t)}], \quad (49a)$$

$$v_{1\bar{q}}(t) = E[\psi_{\bar{q}}^{(0,1,0)}(q(t), \mathbf{x}(t))] = E[x_1(t)\delta_{\bar{q},q(t)}], \quad (49b)$$

$$v_{2\bar{q}}(t) = E[\psi_{\bar{q}}^{(0,0,1)}(q(t), \mathbf{x}(t))] = E[x_2(t)\delta_{\bar{q},q(t)}]. \quad (49c)$$

Applying (22) and (23) to each function $\psi(q, \mathbf{x}) = \psi_{\bar{q}}^{(m)}(q, \mathbf{x})$ yields a set of first order linear differential equations for $\pi_{\bar{q}}(t)$ and $v_{i\bar{q}}(t)$ for $\bar{q} \in \{0, 1, \dots, 6\}$ and $i = 0, 1, 2$. This results in 28 equations, but a number of variables will prove to be trivially zero. As in the LCFS-S system, $\pi_{\bar{q}}(t)$ will converge to the stationary probability $\pi_{\bar{q}}^*$ and each $v_{i\bar{q}}(t)$ will converge to a steady state value $v_{i\bar{q}}^*$ as $t \rightarrow \infty$. The average age Δ_1 can then be calculated from (27). In Appendix C, some additional algebra enables explicit derivation of the closed-form expression of the age Δ_1 in Theorem 3(b).

V. PERFORMANCE EVALUATION

Here we use Theorems 2 and 3 to examine achievable AoI regions for two-user FCFS and LCFS systems. In addition, resource sharing issues for N sources are explored in Section V-C.

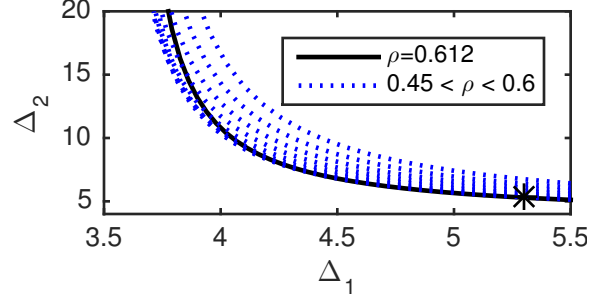


Fig. 7. Age contours (Δ_1, Δ_2) for fixed total load $\rho_1 + \rho_2 = \rho$ for two sources sharing a rate $\mu = 1$ FCFS M/M/1 queue. In the lower right corner, the $\rho = 0.612$ contour is seen to be pareto optimal for $\rho_1 \approx \rho_2$. However, in the upper left corner, $\rho \approx 0.53$ can reduce Δ_1 for constant Δ_2 as $\rho_2 \rightarrow 0$ and $\rho_1 \rightarrow \rho$.

A. M/M/1 FCFS: Two Sources

For a two-user system with normalized service rate $\mu = 1$, Theorem 2 yields the contours of achievable age pairs (Δ_1, Δ_2) for fixed load ρ that are shown in Figure 6. The set of feasible age pairs (Δ_1, Δ_2) is given by the union of all such contours. The lower left “corner” point (marked *) where the sum $\Delta_1 + \Delta_2$ is minimized is obtained at $\rho_1 = \rho_2 = 0.306$, yielding $\Delta_1 = \Delta_2 = 5.30$. It then follows from Theorem 2 that if those two sources were to share a rate $\mu = 2$ server, then each source would obtain average age $\Delta_1 = \Delta_2 = 2.65$. By comparison, if server resources were partitioned and each source employed an individual rate $\mu = 1$ server with optimal load $\rho_1 = 0.531$, Equation (7) will yield $\Delta_1 = 3.48$. Thus we observe a trunking efficiency in having two status-updating sources share a combined service facility.

We note that the $\rho = 0.612$ age contour appears in Figure 6 to be superior to all other age contours. In fact, this contour marks the pareto frontier of achievable ages at the operating point *, corresponding to offered loads $\rho_1 = \rho_2$. However, the optimal load ρ will vary along the pareto frontier. For example, as $\rho_1 \rightarrow \rho$ and $\rho_2 \rightarrow 0$, the $\rho = 0.53$ (the optimal load for a single source) contour offers reduced Δ_1 for fixed Δ_2 . This is shown in Figure 7 where the solid line marks the $\rho = 0.612$ contour while dotted lines mark contours for ρ in the interval $0.45 \leq \rho \leq 0.6$. The general insight for FCFS systems is that multiuser age optimization depends on both the total load ρ and the allocation of load among individual sources.

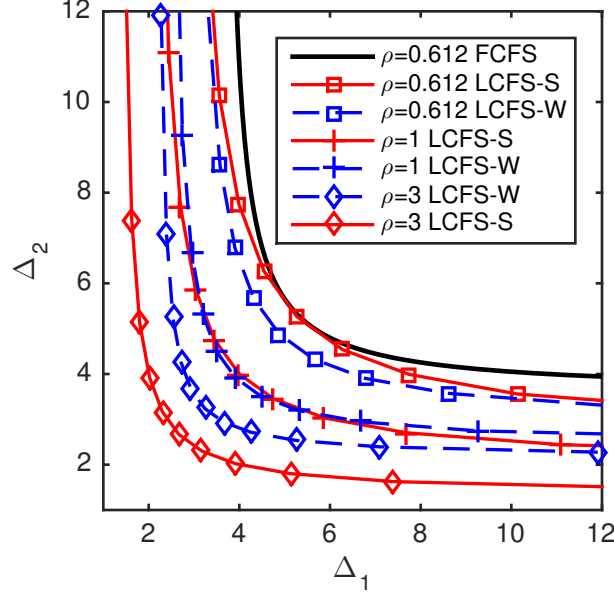


Fig. 8. Update ages of two users under the queuing policies of FCFS, LCFS with preemption in service (LCFS-S), and LCFS with preemption only in waiting (LCFS-W). The service rate is $\mu = 1$.

B. *M/M/1 LCFS: Two Sources*

With $N = 1$ source, some algebra applied to Theorem 3 will verify that LCFS-S (with preemption in service) yields smaller age Δ_1 than LCFS-W (with preemption only in waiting). For $N = 2$ sources, Figure 8 compares the LCFS policies for different choices of the arrival rate ρ . The achievable age pair contour for a given ρ is obtained by varying ρ_1 and ρ_2 such that $\rho_1 + \rho_2 = \rho$. The service rate is $\mu = 1$. We observe that LCFS-W is better than LCFS-S when arrival rates ρ are low but somewhat worse when arrival rates are high. Because this same behavior does not hold for the single source system, we speculate that LCFS-W benefits at low arrival rates from not preempting an update in service with an update from some other source whose update is not currently in service.

For comparison with FCFS, we also plot the FCFS age contour for total load $\rho = 0.612$, which was shown in Figure 6 to be optimal in the neighborhood of $\rho_1 \approx \rho_2$ and near-optimal elsewhere. For the same total offered load $\rho = 0.612$, Figure 8 shows that the age contours obtained under both LCFS policies are better than those obtained under FCFS. To summarize Figure 8, if a system can choose all of ρ , ρ_1 , and ρ_2 , LCFS-S is the policy that minimizes sum age.

In general, the choice of policy is not as straightforward. Figure 9 shows, for $N = 2$ sources and $\mu = 1$, that the policy that minimizes the sum age over points in the cartesian product of $\{0 < \rho_1/\rho \leq 0.5\}$ and $\{0 < \rho < 2\}$. FCFS is the policy of choice for $\rho < 0.4$. It is also the policy of choice for larger $\rho < 1$, however, only when type 1 updates constitute a small enough fraction of ρ . This fraction gets smaller as $\rho \rightarrow 1$. Similarly, LCFS-W is a policy of choice over LCFS-S even for large ρ when the load due to type 1 updates is a small enough fraction of the total load ρ .

C. Multiuser Resource Allocation

We can compare the FCFS and LCFS systems in terms of the sum of ages $\Delta_\Sigma = \sum_{i=1}^N \Delta_i$ when users share the system capacity in fixed proportions such that $\rho_i = \alpha_i \rho$ with $\sum_{i=1}^N \alpha_i = 1$. We note that Theorem 3 implies that the following observations hold for both types of LCFS systems:

- Each user i has age Δ_i that decreases monotonically with total load ρ .
- The sum age Δ_Σ is minimized by equal offered loads $\rho_i = \rho/N$.

In addition, it follows from Theorem 3 that

- Δ_i in a LCFS-W system is strictly less than that under LCFS-S iff

$$\frac{1}{N} \sum_{i=1}^N \alpha_i^{-1} > (1 + \rho) \alpha_W(\rho). \quad (50)$$

The right side of Equation (50) is a nondecreasing $O(\rho)$ function. Thus given N updaters and the proportions α_i in which they share the load, there is a maximum ρ such that LCFS-W is better than LCFS-S. We had observed this for $N = 2$ sources in Figures 8 and 9.

Optimization of the offered load ρ is more complicated in the FCFS system. Theorem 2 implies that for each user i , the value of total load ρ that minimizes the average age Δ_i depends on the proportional load α_i . Load optimization of the FCFS system will depend on a performance metric derived from $\Delta_1, \dots, \Delta_N$. However, it is easy to see that for any given $0 < \rho < 1$, the sum of ages Δ_Σ is minimized when $\alpha_i = 1/N$ for all i . This follows from the fact that for a given ρ , $\frac{\partial^2 \Delta_i}{\partial \rho_i^2} > 0$ for $\rho_i \in (0, \rho)$. Also, as a result, the sum of ages is a convex function of ρ_i , $i = 1, \dots, N$. Further, all the Δ_i are the same function over $\rho_i \in (0, \rho)$.

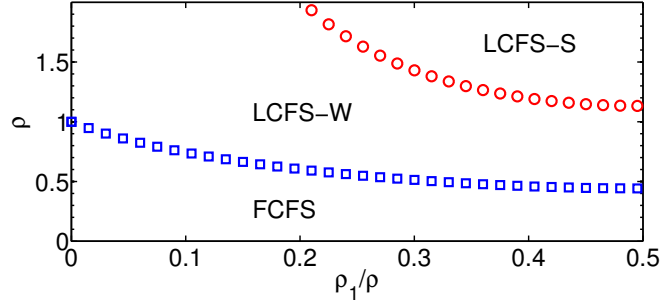


Fig. 9. For the policies of FCFS, LCFS-W, and LCFS-S, we show the region where they are sum age minimizing. FCFS minimizes the sum age for all points $(\rho_1/\rho, \rho)$ that lie below the squares. LCFS-W minimizes the sum age for the points above the squares and to the right of the circles. LCFS-S is sum age minimizing for all points above the circles. The service rate is $\mu = 1$.

To compare the FCFS, LCFS-S and LCFS-W systems, we focus on symmetric systems with $\rho_i = \rho/N$. In this case, each user has identical average age Δ_i given by Theorem 2 or Theorem 3 with $\rho_i = \rho/N$ and $\rho_{-i} = (N-1)\rho/N$. For symmetric LCFS systems, Equation (50) simplifies to

$$N > (1 + \rho)\alpha_W(\rho). \quad (51)$$

Therefore for fixed ρ , LCFS-W outperforms LCFS-S when the number of sources N is large. To further examine this, we now assume a large system with $N \gg 1$ sources. As N becomes large, $\rho_{-i} \rightarrow \rho$ and it follows from (10) that $\alpha_W(\rho)/N \rightarrow 0$. With these limits, it is straightforward to show that Theorems 2 and 3 imply that Δ_i approaches $\Delta_F^N(\rho)$ (FCFS) or $\Delta_S^N(\rho)$ (LCFS-S) or $\Delta_W^N(\rho)$ (LCFS-W) where

$$\Delta_F^N(\rho) = \frac{N}{\mu} \left[\frac{(1 + \rho)\rho^2}{N^3(1 - \rho)^3} + \frac{1}{N(1 - \rho)} + \frac{1}{\rho} \right], \quad (52a)$$

$$\Delta_S^N(\rho) = \frac{N}{\mu} \left[1 + \frac{1}{\rho} \right], \quad (52b)$$

$$\Delta_W^N(\rho) = \frac{N}{\mu} \left[1 + \frac{1}{\rho(1 + \rho)} \right]. \quad (52c)$$

From (52b) and (52c), we see that both LCFS systems have AoI that decreases with total load ρ . By contrast, the FCFS system is subject to the stability constraint $\rho < 1$ and it benefits from matching the load ρ to the number of sources N . Let ρ_N^* minimize $\Delta_F^N(\rho)$ over $0 \leq \rho \leq 1$. Because $\rho < 1$ and N is large, the first term on the right side of (52a) becomes negligible for ρ

near ρ_N^* as N becomes large. Thus ρ_N^* converges to the minimizer of $[N(1 - \rho)]^{-1} + \rho^{-1}$, i.e.,

$$\rho_N^* = \frac{\sqrt{N}}{\sqrt{N} + 1}. \quad (53)$$

It follows that $\Delta_F^N(\rho_N^*) \rightarrow N/\mu$ as N becomes large. Similarly, for the LCFS systems, if ρ grows large as N grows large, then $\Delta_S^N(\rho)$ and $\Delta_W^N(\rho)$ also approach N/μ . In this sense, as the number of users becomes large, all three systems become equivalent. On the other hand, in many settings the offered load ρ may be physically constrained. For example, if each update requires energy for a wireless transmission, then it would be appropriate to compare the FCFS and LCFS systems on an equal power basis. In this spirit, we note that if all three systems operate at offered load $\rho = \rho_N^*$, then $\Delta_W^N(\rho_N^*)/\Delta_F^N(\rho_N^*) \rightarrow 1.5$ and $\Delta_S^N(\rho_N^*)/\Delta_F^N(\rho_N^*) \rightarrow 2$ as N becomes large. Thus, one can argue that FCFS is more efficient than either LCFS discipline for large symmetric systems.

D. Non-cooperative Rate Adaptation

We now examine how sources may individually adapt their updating rates. When source i is a status updater in the presence of “interfering” traffic with aggregate load ρ_{-i} from other sources, it may be in the interest of source i to unilaterally optimize its updating load ρ_i in response to the aggregate other load ρ_{-i} .

For the N -user FCFS system, it was observed [10] that the adaptation

$$\rho_i^*(\rho_{-i}) = \frac{1 - \rho_{-i}}{2} + \frac{[1 - \rho_{-i}]^2}{32} \quad (54)$$

is essentially the same as the best-response normalized load that exactly minimizes Δ_i . It was also noted that since the minima over ρ_i is broad and nearly flat, $\hat{\rho}_i = 0.5(1 - \rho_{-i})$, a rule of thumb that a source should use half the the residual capacity, was a good linear approximation. If each other source is a status updater that selects an update load to minimizes its respective age given the other sources’ update loads, we obtain the synchronous iterative algorithm

$$\rho_i(n+1) = \frac{1 - \rho_{-i}(n)}{2} + \frac{[1 - \rho_{-i}(n)]^2}{32}. \quad (55)$$

This iterative algorithm was shown to work reasonably well for $N = 2$ users as it converges to a fixed point with $\rho_1 = \rho_2 = 0.342$, and corresponding ages $\Delta_1 = \Delta_2 = 5.4390$ [10]. However,

it is easily verified that this iteration is unstable for $N > 2$ users. Similar distributed algorithms $\hat{\rho}_i(n+1) = \omega_N[1 - \rho_{-i}(n)]$, in which each node uses a fraction ω_N (depending on the number of sources N) of the residual capacity can be shown to be stable. However, a network mechanism that sends ω_N to a source i might just as well send the appropriate load ρ_i to that source.

For both LCFS systems, each source i has incentive to generate updates as fast as possible. To be precise, when a source i is subject to a maximum load constraint $\rho_i \leq \bar{\rho}_i$ and other sources offer combined load ρ_{-i} , then source i can decrease its average age Δ_i by unilaterally increasing ρ_i to $\bar{\rho}_i$. The Nash equilibrium of the system is for each node i to operate at maximum updating load $\bar{\rho}_i$. The average age each node will obtain will depend on $\bar{\rho}_i$ and the total updating load $\sum_{k=1}^N \bar{\rho}_k$. This may be a desirable operating point for some nodes but not for others. Moreover, if each node bears some cost for its offered load ρ_i , this operating point may be undesirable even if the resulting age is small.

An alternate approach would be for node i to view its age as a function $\Delta_i(\rho_i, \rho_{-i})$ of its offered load and the interfering load, and to adjust ρ_i to meet a target age constraint $\Delta_i(\rho_i, \rho_{-i}) = \delta_i$. Such an approach shares many commonalities with game-theoretic power control in wireless CDMA systems [32] although discussion of these issues is beyond the scope of this work.

VI. CONCLUSION

We have looked at the problem of multiple sources generating timely status updates at interested recipients. We have employed a simple approach in which the communication network is modeled by an M/M/1 queue with either FCFS service or one of two variants of LCFS service.

For these systems, we have derived and characterized the region of feasible status ages. Our results show that there are nontrivial gains in trunking efficiency when N users share the system capacity. However, achieving these gains appears to require coordinated load balancing of the sources. In particular, high offered load at an FCFS system induces high AoI through queueing delays. A lossy LCFS discipline can mitigate this problem but its packet discarding policy may encourage sources to operate at excessively high offered loads.

We have introduced stochastic hybrid systems as a new way to evaluate AoI in queues with memoryless service. While the SHS method is perhaps tedious when used to derive closed form expressions for AoI, we believe it will facilitate numerical evaluation of substantially more complex systems. In particular, we have employed a restricted form of SHS in this work; a

general SHS with time-varying and state-dependent transition rates also may prove to be useful in further characterizing age processes.

The preliminary insights from these simple models lead us to believe that the age of information represents a new and useful metric for the analysis of status updating systems. Moreover, it should be apparent that many challenges remain in modeling, characterizing and optimizing practical status updating systems.

APPENDIX A

PROOF OF LEMMA 1

The proof of Lemma 1 relies on the following basic properties of Poisson processes and exponential random variables.

Lemma 2: Let X_1 and X_2 be independent exponential random variables with $E[X_i] = 1/\alpha_i$. Let $V = X_2 - X_1$.

- (a) $P[X_1 < X_2] = \alpha_1/(\alpha_1 + \alpha_2)$.
- (b) Given $X_1 < X_2$, X_1 and V are conditionally independent and have conditional exponential probability density functions (PDFs)

$$\begin{aligned} f_{X_1|X_1 < X_2}(x) &= (\alpha_1 + \alpha_2)e^{-(\alpha_1 + \alpha_2)x}, & x \geq 0, \\ f_{V|X_1 < X_2}(v) &= \alpha_2 e^{-\alpha_2 v}, & v \geq 0. \end{aligned}$$

Lemma 3: Given a rate λ Poisson process $N(t)$ and an independent exponential (α) random variable X , the number of arrivals $N(X)$ in the interval $[0, X]$ has the geometric PMF

$$P_{N(X)}(n) = (1 - \gamma)\gamma^n, \quad n \geq 0,$$

with $\gamma = \lambda/(\alpha + \lambda)$.

To prove Lemma 1, let Y_j , W_j and T_j to refer to the interarrival time, waiting time and system time of the j th packet of source i . We now examine W_j via the partition

$$B_j = \{Y_j < T_{j-1}\}, \quad L_j = \{T_{j-1} < Y_j\}. \quad (56)$$

That is, B_j denotes the event that the j th interarrival time for source i is brief, specifically, less than the system time of the preceding packet from source i , and L_j is the complementary event

that Y_j is long. With the partition $\{B_j, L_j\}$, we write

$$\mathbb{E}[Y_j W_j] = \mathbb{E}[Y_j W_j | L_j] \mathbb{P}[L_j] + \mathbb{E}[Y_j W_j | B_j] \mathbb{P}[B_j]. \quad (57)$$

Since source i packets and packets from other sources have identical exponential (μ) service times, the combined queue is just M/M/1 with offered load $\rho = \rho_i + \rho_{-i}$. In steady state, the system time T_{j-1} has the exponential $(\mu - \lambda)$ PDF

$$f_T(t) = (\mu - \lambda)e^{-(\mu - \lambda)t}, \quad t \geq 0. \quad (58)$$

Furthermore, T_{j-1} , which depends on packets (and their service times) that arrived prior to packet $j - 1$, is independent of Y_j . Given B_j , packet $j - 1$ is still in the system when packet j is generated. The waiting time W_j depends on both the residual system time $T_{j-1} - Y_j$ and also on the workloads of packets from other sources that arrive during the source i interarrival period of length Y_j . Specifically, let $M = N_{-i}(Y_j)$ denote the number of other-source (i.e. not source i) packets that arrive during the source i interarrival period and let S_1, S_2, \dots, S_M denote their service requirements. As these packets are all queued between packets $j - 1$ and j ,

$$W_j = (T_{j-1} - Y_j) + \sum_{k=1}^M S_k. \quad (59)$$

This implies $\mathbb{E}[Y_j W_j | B_j] = E_1 + E_2$ where

$$E_1 = \mathbb{E}[Y_j(T_{j-1} - Y_j) | B_j], \quad (60a)$$

$$E_2 = \mathbb{E}\left[Y_j \sum_{k=1}^M S_k | B_j\right]. \quad (60b)$$

By Lemma 2(b),

$$\begin{aligned} E_1 &= \mathbb{E}[(T_{j-1} - Y_j) | B_j] \mathbb{E}[Y_j | B_j] \\ &= \frac{1}{\mu - \lambda} \left(\frac{1}{\lambda_i + (\mu - \lambda)} \right) = \frac{1}{\mu^2(1 - \rho)(1 - \rho_{-i})}. \end{aligned} \quad (61)$$

For the second term, iterated expectation yields

$$E_2 = \int_0^\infty \mathbb{E}\left[Y_j \sum_{k=1}^M S_k | B_j, Y_j = y\right] f_{Y_j|B_j}(y) dy$$

$$= \int_0^\infty \mathbb{E} \left[y \sum_{k=1}^M S_k | Y_j = y \right] f_{Y_j|B_j}(y) dy. \quad (62)$$

Given that $Y_j = y$, $M = N_{-i}(Y_j) = N_{-i}(y)$ is the number of other-source arrivals in a period of length y and is Poisson with conditional expectation $\mathbb{E}[M|Y_j = y] = \lambda_{-i}y$. In addition, each S_k is an exponential (μ) random variable, independent of Y_j , implying $\mathbb{E}[S_k|Y_j = y] = 1/\mu$. This implies

$$\mathbb{E} \left[y \sum_{k=1}^M S_k | Y_j = y \right] = y \mathbb{E}[M|Y_j = y] \mathbb{E}[S_k|Y_j = y] = y(\lambda_{-i}y)(1/\mu) = \rho_{-i}y^2. \quad (63)$$

By Lemma 2, Y_j given B_j is an exponential (α) random variable with $\alpha = \lambda_i + (\mu - \lambda_i - \lambda_{-i}) = \mu - \lambda_{-i}$. This implies

$$E_2 = \rho_{-i} \int_0^\infty y^2 \alpha e^{-\alpha y} dy = \frac{2\rho_{-i}}{\alpha^2} = \frac{2\rho_{-i}}{\mu^2(1 - \rho_{-i})^2}. \quad (64)$$

It follows from (61) and (64) that

$$\mathbb{E}[Y_j W_j | B_j] = \frac{1}{\mu^2} \left[\frac{2\rho_{-i}}{(1 - \rho_{-i})^2} + \frac{1}{(1 - \rho_{-i})(1 - \rho)} \right]. \quad (65)$$

Given event L_j , packet $j - 1$ has departed the system prior to the arrival of packet j . In this case, the waiting time for packet j depends on the number of other-source packets in the system when packet j arrives. To characterize this, we now let M denote the number of other-source packets in the system at the departure instant of packet $j - 1$. Since the queue is FCFS, M is the number of other-source packets that arrived and were queued during the system time T_{j-1} of packet $j - 1$. Given T_{j-1} is exponential and independent of Y_j , Lemma 2(b) tells us that T_{j-1} given L_j is conditionally an exponential (α) random variable with $\alpha = (\mu - \lambda) + \lambda_i = \mu - \lambda_{-i}$. As T_{j-1} is independent of the subsequent Poisson arrivals of the other sources, Lemma 3 implies that M has the geometric PMF

$$P_M(m) = (1 - \gamma)\gamma^m, \quad m \geq 0, \quad (66)$$

where $\gamma = \lambda_{-i}/(\alpha + \lambda_{-i}) = \rho_{-i}$.

From (66), we see at the departure instant of packet $j - 1$ that M is described by the stationary distribution of an M/M/1 queue serving only other-source packets at rate λ_{-i} . Going forward

from this instant, we wait an additional time $Y_j - T_{j-1}$ for packet j from source i . In this time period, there may be either arrivals or departures of other-source packets. Nevertheless, as the queue holds zero source i packets, the operation of the queue is identical to an M/M/1 queue for just other-source packets. At all times up to the arrival of packet j , the number of other-source packets in the queue remains stochastically identical to M . If the k th queued other-source packet has service requirement S_k , then $W_j = \sum_{k=1}^M S_k$ and $E[W_j|L_j] = E[M]/\mu$. It follows that when packet j does arrive, the number of queued packets M and the service times S_k are independent of both the additional delay $Y_j - T_{j-1}$ until the arrival of packet j and the prior system time T_{j-1} . This implies

$$\begin{aligned} E[Y_j W_j | L_j] &= E[(T_{j-1} + (Y_j - T_{j-1})) W_j | L_j] \\ &= E[T_{j-1} + (Y_j - T_{j-1}) | L_j] E[W_j | L_j] \\ &= \left(\frac{1}{\mu - \lambda_{-i}} + \frac{1}{\lambda_i} \right) \left(\frac{\rho_{-i}}{\mu(1 - \rho_{-i})} \right). \end{aligned} \quad (67)$$

Next we recall from Lemma 2 that independence of T_{j-1} and Y_j implies $P[B_j] = \rho_i/(1 - \rho_{-i})$. Combining this fact with (57), (65), and (67), some algebra yields the claim.

APPENDIX B

THEOREM 3(A): PROOF COMPLETION

Note that $T = T_j$ is the time that packet j from source i spends in service. This packet completes service (is not preempted) and hence no new arrivals occur during the interval S_L . The distribution of T is that of the time to service completion, say X_μ , after packet j arrives, conditioned on X_μ being smaller than the time to the next packet arrival, say X_λ , from any source. Thus $P[T > z] = P[X_\mu > z | X_\mu < X_\lambda]$. By Lemma 2(b), T_j is exponential $(\lambda + \mu)$ and $E[T] = 1/(\lambda + \mu)$.

Now we calculate the moments of $D = D_j$. From (11) we know that D is a random sum of random variables B_k , $1 \leq k \leq L$. Also, D ends with the departure of a source i update packet. Since the arrival rate λ is the sum of rates of independent Poisson sources, the probability that any block B_k ends in the departure of the update packet of source i is λ_i/λ . Note that D consists of $L = l$ blocks if $l - 1$ consecutive blocks end in departures of other-source packets, followed

by block l ending in a source i departure. Thus,

$$P[L = l] = (1 - q)^{l-1} q, \quad l \geq 1, \quad (68)$$

where $q = \lambda_i / \lambda$. Note that $B_k = X'_k + S_k$, where X'_k is an idle period and S_k is the server busy period. During the busy period a random number of packets in service may be preempted, but the service rate (i.e., the instantaneous departure rate) is μ throughout the busy period. Thus the busy period S_k is memoryless and is independent of the number of arrivals during it that get preempted and the source whose packet departs at its end. From these observations and given Poisson arrivals of rate λ , we can write

$$E[X'_k] = \frac{1}{\lambda}, \quad E[S_k] = \frac{1}{\mu}, \quad \text{and} \quad E[B_k] = \frac{1}{\lambda} + \frac{1}{\mu}. \quad (69)$$

The memoryless nature of the arrival and service processes also implies that each B_k is independent of L . Using this fact and equations (11), (68) and (69), we can write

$$E[D] = E[L] E[B_k] = \frac{\mu + \lambda}{\lambda_i \mu}. \quad (70)$$

To calculate $E[D^2]$, let the random variable B be stochastically identical to block lengths B_k , $k = 1, \dots, L$. Using arguments we used to calculate $E[D]$, and noting that B_k and $B_{k'}$ are independent for $k \neq k'$, we can write

$$E[D^2] = E[L] E[B^2] + E[L(L-1)] (E[B])^2. \quad (71)$$

Also note that the idle period X'_k and busy period S_k that constitute B_k are mutually independent. This implies

$$E[B^2] = \frac{2}{\lambda^2} + \frac{2}{\mu^2} + \frac{2}{\lambda\mu}. \quad (72)$$

Using equations (68), (69) and (72), we can write (71) as

$$E[D^2] = 2 \frac{\lambda}{\lambda_i} \left(\frac{\lambda}{\lambda_i} \left[\frac{1}{\lambda} + \frac{1}{\mu} \right]^2 - \frac{1}{\lambda\mu} \right). \quad (73)$$

Applying the moments $E[T]$, $E[D]$ and $E[D^2]$ to equation (16) yields Theorem 3(a).

APPENDIX C

THEOREM 3(B): PROOF COMPLETION

From (22), we calculate $L\psi_{\bar{q}}^{(m)}$ for each m and $\bar{q} \in \mathcal{Q}$:

$$L\psi_{\bar{q}}^{(m)}(q, \mathbf{x}) = m_0 x_0^{m_0-1} x_1^{m_1} x_2^{m_2} \delta_{0,q} + \mu \Lambda_{\bar{q}}^{(m)}(q, \mathbf{x}) \quad (74)$$

where $\Lambda_{\bar{q}}^{(m)}(q, \mathbf{x})$ is given by (29). For the LCFS-W system, it follows from Table II that

$$\Lambda_0^{(m)} = -\rho \mathbf{x}^m \delta_{0,q} + [x_0 - x_1, 0, 0]^m \delta_{1,q} + [x_0, 0, 0]^m \delta_{4,q}, \quad (75a)$$

$$\Lambda_1^{(m)} = \rho_1 [x_0, x_0, 0]^m \delta_{0,q} - \beta \mathbf{x}^m \delta_{1,q} + [x_0 - x_1, x_2, 0]^m \delta_{2,q} + [x_0, x_2, 0]^m \delta_{6,q}, \quad (75b)$$

$$\Lambda_2^{(m)} = \rho_1 [x_0, x_1, x_0 - x_1]^m (\delta_{1,q} + \delta_{2,q} + \delta_{3,q}) - \beta \mathbf{x}^m \delta_{2,q} \quad (75c)$$

$$\Lambda_3^{(m)} = \rho_2 [x_0, x_1, 0]^m (\delta_{1,q} + \delta_{2,q}) - \beta_1 \mathbf{x}^m \delta_{3,q}, \quad (75d)$$

$$\Lambda_4^{(m)} = \rho_2 [x_0, 0, 0]^m \delta_{0,q} + [x_0 - x_1, 0, 0]^m \delta_{3,q} - \beta \mathbf{x}^m \delta_{4,q} + [x_0, 0, 0]^m \delta_{5,q}, \quad (75e)$$

$$\Lambda_5^{(m)} = \rho_2 [x_0, 0, 0]^m (\delta_{4,q} + \delta_{6,q}) - \beta_1 \mathbf{x}^m \delta_{5,q}, \quad (75f)$$

$$\Lambda_6^{(m)} = \rho_1 [x_0, 0, x_0]^m (\delta_{4,q} + \delta_{5,q} + \delta_{6,q}) - \beta \mathbf{x}^m \delta_{6,q}. \quad (75g)$$

Applying (23) to $\psi(q(t), \mathbf{x}(t)) = \psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))$, it follows from (74) that

$$\frac{d \mathbb{E}[\psi_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))]}{dt} = m_0 \mathbb{E}[\mathbf{x}^{(m_0-1, m_1, m_2)}(t) \delta_{\bar{q}, q(t)}] + \mu \mathbb{E}[\Lambda_{\bar{q}}^{(m)}(q(t), \mathbf{x}(t))]. \quad (76)$$

For $m = (0, 0, 0)$, (47) and (76) that the state probability vector $\pi(t) = [\pi_0(t) \ \dots \ \pi_6(t)]'$ has component derivatives $\dot{\pi}_{\bar{q}}(t) = d\pi_{\bar{q}}(t)/dt$ that satisfy

$$\dot{\pi}_{\bar{q}}(t) = \mu \mathbb{E}[\Lambda_{\bar{q}}^{(0,0,0)}(q(t), \mathbf{x}(t))]. \quad (77)$$

Furthermore, for $m = (0, 0, 0)$, (75) reduces to

$$\Lambda_0^{(0,0,0)} = -\rho \delta_{0,q(t)} + \delta_{1,q(t)} + \delta_{4,q(t)}, \quad (78a)$$

$$\Lambda_1^{(0,0,0)} = \rho_1 \delta_{0,q(t)} - \beta \delta_{1,q(t)} + \delta_{2,q(t)} + \delta_{6,q(t)}, \quad (78b)$$

$$\Lambda_2^{(0,0,0)} = \rho_1 \delta_{1,q(t)} - \beta \delta_{2,q(t)} + \rho_1 (\delta_{2,q(t)} + \delta_{3,q(t)}), \quad (78c)$$

$$\Lambda_3^{(0,0,0)} = \rho_2 (\delta_{1,q(t)} + \delta_{2,q(t)}) - \beta_1 \delta_{3,q(t)}, \quad (78d)$$

$$\Lambda_4^{(0,0,0)} = \rho_2 \delta_{0,q(t)} + \delta_{3,q(t)} - \delta_{4,q(t)} + \delta_{5,q(t)}, \quad (78e)$$

$$\Lambda_5^{(0,0,0)} = \rho_2 \delta_{4,q(t)} - \beta_1 \delta_{5,q(t)} + \rho_2 \delta_{6,q(t)}, \quad (78f)$$

$$\Lambda_6^{(0,0,0)} = \rho_1 (\delta_{4,q(t)} + \delta_{5,q(t)}) - \beta_2 \delta_{6,q(t)}. \quad (78g)$$

Thus (77) implies that $\pi(t)$ satisfies $\dot{\pi}(t) = \mu \mathbf{R} \pi(t)$ with

$$\mathbf{R} = \begin{bmatrix} -\rho & 1 & 0 & 0 & 1 & 0 & 0 \\ \rho_1 & -\beta & 1 & 0 & 0 & 0 & 1 \\ 0 & \rho_1 & -\beta_2 & \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & \rho_2 & -\beta_1 & 0 & 0 & 0 \\ \rho_2 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & \rho_2 & -\beta_1 & \rho_2 \\ 0 & 0 & 0 & 0 & \rho_1 & \rho_1 & -\beta_2 \end{bmatrix}. \quad (79)$$

Setting $\dot{\pi}(t) = 0$ yields the stationary probabilities

$$\pi^* = C_\pi \begin{bmatrix} 1 & \rho_1 & \rho_1^2 & \rho_1 \rho_2 & \rho_2 & \rho_2^2 & \rho_1 \rho_2 \end{bmatrix}' \quad (80)$$

where $C_\pi^{-1} = \beta + \rho^2 = 1 + \rho + \rho^2$. We note that π^* is simple (relative to the apparent complexity of the Markov chain in Figure 5) because the discrete state distinguishes between source 1 and 2 packets even though the service times are identical for the sources.

For $m = (1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, (49) and (76) imply $v_{0\bar{q}}(t)$, $v_{1\bar{q}}(t)$, and $v_{2\bar{q}}(t)$ have derivatives satisfying

$$\dot{v}_{0\bar{q}}(t) = \mathbb{E}[\delta_{\bar{q},q(t)}] + \mu \mathbb{E}[\Lambda_{\bar{q}}^{(1,0,0)}(q(t), \mathbf{x}(t))], \quad (81a)$$

$$\dot{v}_{1\bar{q}}(t) = \mu \mathbb{E}[\Lambda_{\bar{q}}^{(0,1,0)}(q(t), \mathbf{x}(t))], \quad (81b)$$

$$\dot{v}_{2\bar{q}}(t) = \mu \mathbb{E}[\Lambda_{\bar{q}}^{(0,0,1)}(q(t), \mathbf{x}(t))]. \quad (81c)$$

For $m = (1, 0, 0)$, the expectation of (75) is

$$\mathbb{E}[\Lambda_0^{(1,0,0)}] = -\rho v_{00} + v_{01} - v_{11} + v_{04}, \quad (82a)$$

$$\mathbb{E}[\Lambda_1^{(1,0,0)}] = \rho_1 v_{00} - \beta v_{01} + v_{02} - v_{12} + v_{06}, \quad (82b)$$

$$\mathbb{E}[\Lambda_2^{(1,0,0)}] = \rho_1 (v_{01} + v_{02} + v_{03}) - \beta v_{02} \quad (82c)$$

$$\mathbb{E}[\Lambda_3^{(1,0,0)}] = \rho_2(v_{01} + v_{02}) - \beta_1 v_{03}, \quad (82d)$$

$$\mathbb{E}[\Lambda_4^{(1,0,0)}] = \rho_2 v_{00} + v_{03} - v_{13} - \beta v_{04} + v_{05}, \quad (82e)$$

$$\mathbb{E}[\Lambda_5^{(1,0,0)}] = \rho_2(v_{04} + v_{06}) - \beta_1 v_{05}, \quad (82f)$$

$$\mathbb{E}[\Lambda_6^{(1,0,0)}] = \rho_1(v_{04} + v_{05} + v_{06}) - \beta v_{06}. \quad (82g)$$

For $m = (0, 1, 0)$, (75) yields

$$\mathbb{E}[\Lambda_0^{(0,1,0)}] = -\rho v_{10}, \quad (83a)$$

$$\mathbb{E}[\Lambda_1^{(0,1,0)}] = \rho_1 v_{00} - \beta v_{11} + v_{22} + v_{26}, \quad (83b)$$

$$\mathbb{E}[\Lambda_2^{(0,1,0)}] = \rho_1(v_{11} + v_{12} + v_{13}) - \beta v_{12} \quad (83c)$$

$$\mathbb{E}[\Lambda_3^{(0,1,0)}] = \rho_2(v_{11} + v_{12}) - \beta_1 v_{13}, \quad (83d)$$

$$\mathbb{E}[\Lambda_4^{(0,1,0)}] = -\beta v_{14}, \quad (83e)$$

$$\mathbb{E}[\Lambda_5^{(0,1,0)}] = -\beta_1 v_{15}, \quad (83f)$$

$$\mathbb{E}[\Lambda_6^{(0,1,0)}] = -\beta v_{16}. \quad (83g)$$

We see from (81b) and (83a) that v_{10} , v_{14} , v_{15} and v_{16} each obey a differential equation of the form $\dot{v}_{ij}(t) = -\alpha_{ij}v_{ij}(t)$ for a constant $\alpha_{ij} > 0$. This is a consequence of each of these $v_{ij}(t)$ being trivially zero at all times t . Inspection of the Markov chain of Figure 5 will show that v_{20} , v_{21} , v_{23} , v_{24} , and v_{25} are also all trivially zero.

For $m = (0, 0, 1)$, we employ (81c) to obtain equations associated with the nontrivial variables v_{22} and v_{26} :

$$\mathbb{E}[\Lambda_2^{(0,0,1)}] = \rho_1(v_{01} + v_{02} + v_{03} - v_{11} - v_{12} - v_{13}) - \beta v_{22}, \quad (84a)$$

$$\mathbb{E}[\Lambda_6^{(0,0,1)}] = \rho_1(v_{04} + v_{05} + v_{06}) - \beta v_{26}. \quad (84b)$$

We gather the nontrivial variables in the vectors

$$\mathbf{v}_0 = \begin{bmatrix} v_{00} & v_{01} & \cdots & v_{06} \end{bmatrix}', \quad (85)$$

$$\mathbf{v}_1 = \begin{bmatrix} v_{11} & v_{12} & v_{13} & v_{22} & v_{26} \end{bmatrix}' \quad (86)$$

and we use (81), (82), (83), and (84) to write

$$\begin{bmatrix} \dot{\mathbf{v}}_0(t) \\ \dot{\mathbf{v}}_1(t) \end{bmatrix} = \mu \begin{bmatrix} \mathbf{A}_{00} & \mathbf{A}_{01} \\ \mathbf{A}_{10} & \mathbf{A}_{11} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0(t) \\ \mathbf{v}_1(t) \end{bmatrix} + \begin{bmatrix} \pi(t) \\ \mathbf{0} \end{bmatrix} \quad (87)$$

where

$$\mathbf{A}_{00} = \begin{bmatrix} -\rho & 1 & 0 & 0 & 1 & 0 & 0 \\ \rho_1 & -\beta & 1 & 0 & 0 & 0 & 1 \\ 0 & \rho_1 & -\beta_2 & \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & \rho_2 & -\beta_1 & 0 & 0 & 0 \\ \rho_2 & 0 & 0 & 1 & -\beta & 1 & 0 \\ 0 & 0 & 0 & 0 & \rho_2 & -\beta_1 & \rho_2 \\ 0 & 0 & 0 & 0 & \rho_1 & \rho_1 & -\beta_2 \end{bmatrix}, \quad \mathbf{A}_{01} = - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (88a)$$

$$\mathbf{A}_{10} = \rho_1 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{11} = \begin{bmatrix} -\beta & 0 & 0 & 1 & 1 \\ \rho_1 & -\beta_2 & \rho_1 & 0 & 0 \\ \rho_2 & \rho_2 & -\beta_1 & 0 & 0 \\ -\rho_1 & -\rho_1 & -\rho_1 & -\beta & 0 \\ 0 & 0 & 0 & 0 & -\beta \end{bmatrix}. \quad (88b)$$

As $t \rightarrow \infty$, $\dot{\mathbf{v}}_i(t) \rightarrow 0$, $\dot{\pi}(t) \rightarrow \pi^*$ and $\mathbf{v}_i(t) \rightarrow \mathbf{v}_i^*$ for $i = 1, 2$. It then follows from (87) that

$$\mathbf{v}_1^* = -\mathbf{A}_{11}^{-1} \mathbf{A}_{10} \mathbf{v}_0^*, \quad (89)$$

and that

$$(\mathbf{A}_{00} - \mathbf{C}_{00}) \mathbf{v}_0^* = \pi^* / \mu \quad (90)$$

where $\mathbf{C}_{00} = \mathbf{A}_{01} \mathbf{A}_{11}^{-1} \mathbf{A}_{10}$. Some algebra will show that

$$\mathbf{C}_{00} = \frac{\rho_1}{\beta(\rho_1 + \beta)} \mathbf{c}_{00} \begin{bmatrix} \beta & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (91)$$

with $\mathbf{c}_{00} = \begin{bmatrix} 1 & \rho_1 & 0 & 0 & \rho_2 & 0 & 0 \end{bmatrix}'$. The zero rows of \mathbf{A}_{00} and \mathbf{c}_{00} corresponding to v_{02}^* , v_{03}^* ,

v_{05}^* and v_{06}^* imply

$$\begin{bmatrix} v_{02}^* \\ v_{03}^* \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} \left(\frac{C_\pi \rho_1}{\mu} + v_{01}^* \right), \quad (92a)$$

$$\begin{bmatrix} v_{05}^* \\ v_{06}^* \end{bmatrix} = \begin{bmatrix} \rho_2 \\ \rho_1 \end{bmatrix} \left(\frac{C_\pi \rho_2}{\mu} + v_{04}^* \right). \quad (92b)$$

This enables us to simplify (90). Specifically, defining $\hat{\mathbf{v}} = [v_{00}^* \ v_{01}^* \ v_{04}^*]'$, (90) reduces to

$$\begin{bmatrix} \rho_1 + \rho & -1 & -1 \\ -\rho_1 & \beta & -\rho_1 \\ -\rho_2 & -\rho_2 & \rho_1 + \beta_1 \end{bmatrix} \hat{\mathbf{v}} = \frac{C_\pi}{\mu \beta^2} \begin{bmatrix} \beta(\rho_1 + \beta) - \rho_1 \rho^2 \\ \rho_1 [\beta^2(\rho_1 + \beta) - \rho_1 \rho^2] \\ \rho_2 [\beta^2(\rho_1 + \beta) - \rho_1 \rho^2] \end{bmatrix}. \quad (93)$$

Solving for $\hat{\mathbf{v}}$, we obtain

$$\begin{bmatrix} v_{00}^* \\ v_{01}^* \\ v_{04}^* \end{bmatrix} = \frac{C_\pi}{\mu \rho_1 \beta^2} \begin{bmatrix} \beta \beta_1 + \rho \beta^2 - \rho_1 \rho^2 \\ \rho_1 [\beta + \rho \beta^2 + \rho_1 (\beta^2 - \rho^2)] \\ \rho_2 [\beta + \rho \beta^2 + \rho_1 (\beta^2 - \rho^2)] \end{bmatrix}. \quad (94)$$

It follows from (27) and (92) that

$$\Delta_1 = \sum_{\bar{q}=0}^7 v_{0\bar{q}}^* = v_{00}^* + \beta v_{01}^* + \beta v_{04}^* + \frac{\rho^2 C_\pi}{\mu}. \quad (95)$$

The claim of Theorem 3(b) then follows from (94).

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